Lecture courses

A. Parshin. Representations theory of adelic groups and arithmetic

We will present a survey of the representations theory for discrete nilpotent groups and it's applications to the adelic groups of two-dimensional schemes. Reference: arXiv:1012.0486

M. Tsfasman. Codes and sphere packings for a number theorist

Simple combinatorial objects like error-correcting codes and geometric objects like packings of equal spheres happen to be linked to many interesting questions of number theory and arithmetic geometry. I shall give an introduction into codes and packings, then describe some existing applications of number theory to both, and then discuss open questions and not yet well-enough-developed asymptotic theory of number and function fields.

G. Wüstholz. The analytic subgroup theorem, logarithmic forms and periods

In this series of lectures we shall give an overview about modern developments in transcendence theory originating in Hilbert's 7^{th} solved independently by A. O. Gelfond and Th. Schneider on 1934. We shall in the first lecture give an exposition on the Analytic Subgroup Theorem and the theory of multip0licity estimates which are behind. The second lecture deals with applications in the theory of logarithmic forms. This will cover the isogeny theorem which gives an alternative transcendental approach to the Tate conjecture. As is well known that Tate conjecture implies the Shafarevich conjecture from which, by a beautiful device found by Kodaira and Parshin, a proof of the Mordell conjecture can be obtained, and this gives Falting's Theorem on rational points on curves. We shall also touch the André–Oort conjecture in the context of transcendence properties of hypergeometric functions. The last lecture is devoted to a conjecture of Leibniz as described by Arnold in his book on Huygens and Barrow, Newton and Hook. We shall formulate a modern version of the conjecture, relate it to a conjecture of Kontsevich on motives and to a generalization of the famous Schanuel conjecture.

Talks

V. Abrashkin. A semi-stable case of the Shafarevich Conjecture

Suppose F is the quotient field of the ring of Witt vectors with coefficients in an algebraically closed field k of odd characteristic p. We construct an integral theory of p-adic semi-stable representations of the absolute Galois group of F with Hodge–Tate weights from $[0, p)$. This modification of Breuil's theory results in the following application in the spirit of the Shafarevich Conjecture. If Y is a projective algebraic variety over rational numbers with good reduction away from 3 and semi-stable reduction modulo 3, then for the Hodge numbers of the complexification Y_C of Y it holds $h^2(Y_C) = h^{1,1}(Y_C)$.

B. Doran. A little geometric class field theory and a question of Witten

C. Fuchs. Integral solutions of separated-variables equations

In my talk I shall explain how to explicitly characterize the finiteness of the set of integral solutions of a Diophantine equation with separated variables, i.e. an equation of the form $f(x) = g(y)$ with f, g polynomials having integral coefficients, a result known as the Bilu–Tichy criterion. The proof uses the geometric description of the situation and ultimately follows as an application of Siegel's theorem. Afterwards, I shall discuss which additional information one can get if the equation has infinitely many solutions. These general statements will then be applied to concrete equations where Stirling numbers are involved. At the end of the talk, if time permits, I shall also mention some effective results for some (very) special cases.

N. Giansiracusa. Conformal blocks divisors and points on curves

I'll discuss a relation between certain divisors on the moduli space of stable rational pointed curves arising in conformal field theory, and geometric invariant theory quotients generically parameterizing configurations of points on Veronese curves. This correspondence shows, in particular, that a symmetry in the representation theory of the special linear group can be viewed as a form of Gale duality first proven by Goppa in the context of algebraic coding theory.

P. Habegger. Some results beyond the conjectures of André–Oort and Manin–Mumford

The Manin–Mumford Conjecture, a theorem of Raynaud, governs the distribution of torsion points on subvarieties of abelian varieties with respect to the Zariski topology. The conceptually related André–Oort Conjecture describes the special points on subvarieties of Shimura varieties. It was proved by Klingler, Ullmo, and Yafaev under the Generalized Riemann Hypothesis. In this talk I will describe a recent result on a combination of these two conjectures in the "mixed" setting. I will also report on joint work with Jonathan Pila on unlikely intersections beyond the André–Oort Conjecture in a product of modular curves.

R. von Känel. An effective proof of the hyperelliptic Shafarevich conjecture

Let K be a field, S be a finite set of places of K and let $g \geq 1$ be an integer. The Shafarevich conjecture says that there are only finitely many K-isomorphism classes of curves over K of genus g with good reduction outside S. This was proved by Faltings in 1983. An effective version of the conjecture would imply inter alia the effective Mordell and the abc conjecture. In the talk we give an effective version of the Shafarevich conjecture for hyperelliptic curves and discuss some applications.

M. Korolev. On Gram's Law in the Theory of the Riemann Zeta Function

The report is devoted to the distribution of the ordinates of non-trivial zeros of the Riemann zeta-function and a so-called Gram's law. The main purpose is to reconstruct the proof of one Selberg's theorem announced in 1946.

A. Kresch. Strong approximation and integral Brauer-Manin obstruction

After quickly reviewing the Brauer–Manin obstruction for rational points on a complete variety over a number field we will discuss the setting of open varieties where consideration of the Brauer group leads to an obstruction to strong approximation. We describe some examples and results, including cases where the Brauer–Manin obstruction to strong approximation is known to be the only one.

L. Kühne. On effectivity and uniformity in a result of André–Oort type

The André–Oort Conjecture (AOC) states that the irreducible components of the Zariski closure of a set of special points in a Shimura variety are special subvarieties. Here, a special variety is an irreducible component of the image of a sub-Shimura variety by a Hecke correspondence. In our talk, we plan to discuss a rarely known approach to the André–Oort Conjecture (AOC) that goes back to Yves André himself. Before the recent model-theoretic proofs of the AOC in certain cases by Pila André's proof was the only known unconditional proof of the AOC for a non-trivial Shimura variety. In our talk, we spot light on some recent improvements and additions to André's techniques, which enable us to give an effective proof of the AOC in the case of a product of two modular curves. Furthermore, we discuss the aspect of uniform bounds on the number of special points on a non-special curve in some detail.

Yu. Nesterenko. Some identities of Ramanujan type

We will discuss some algebraically independent functions connected to Eisenstein series and having degeneracy of the transcendence degree of the field generated by their values. This degeneracy is a consequence of some quasi-modular identities that generalize the classical identities of S. Ramanujan and E. Grosswald. Values of these functions at the point i are connected to Riemann zeta-values. For example the following classical equality of M. Lerch holds

$$\zeta(3) = \frac{7\pi^3}{180} - 2\sum_{n=1}^{\infty} \sigma_{-3}(n) e^{-2\pi n}, \qquad \sigma_{-3}(n) = \sum_{d|n} d^{-3}.$$

D. Osipov. Categorical central extensions and reciprocity laws on algebraic surfaces

We will describe how various reciprocity laws on algebraic surfaces can be obtained by means of two-dimensional adelic theory and of calculations of some generalized commutators in categorical central extensions of multiplicative groups of two-dimensional local fields. The talk is based on joint results with Xinwen Zhu.

I. Rezvyakova. An additive type problem for the Fourier coefficients of automorphic cusp forms and its application to the zeros of the associated L-functions on the critical line

In our talk we give a non-trivial estimate for the analogue of the additive Dirichlet divisor problem for the Fourier coefficients coming from an automorphic cusp form for the group $\Gamma_0(D)$ with an arbitrary character. As a result, we obtain that a positive proportion of non-trivial zeros are on the critical line for L-functions associated with such automorphic forms.

S. Rybakov. Constructible DG-modules over the de Rham algebra

Positselski proved that the unbounded derived category of quasi-coherent D-modules on a smooth algebraic variety X is equivalent to a so-called coderived category of quasi-coherent DG-modules over the de Rham algebra of X. First

I will explain how to work with the coderived category in question. Then I introduce constructible DG-modules. They form a nice subcategory of the coderived category. It turns out that constructible DG-modules correspond to holonomic D-modules under Positselski equivalence.

I. Shkredov. Collinear triples, multiplicative subgroups and convex sets

Suppose that we have a finite set of points and lines on the plane. A classical theorem of Szemeredi–Trotter allows us to bound the number of incidences between such points and lines. It turns out that in some problems of additive combinatorics other quantities, namely, the number of collinear points and its analogies, play an important role. Using our method we obtain some applications to the problem of finding lower bounds for the cardinality of subsets of multiplicative subgroups in Z/pZ and also convex subsequences of real numbers.

M. Wang. Zolotarev's fractions revisited

In the formal dictionary between trigonometric functions and elliptic functions, Chebyshev polynomial goes to the elliptic rational function which was first studied by Zolotarev. We shall discuss both old and new ways of the construction of those special functions, together with their applications to arithmetic and algebraic dynamics.

A. Zykin. Asymptotic properties of zeta functions

In this talk we will present the results concerning the asymptotic properties of zeta functions of global fields and varieties over them. We will discuss in more detail the explicit versions of the Brauer–Siegel type theorems and the results on the distribution of zeroes of L-functions of elliptic curves and modular forms. In the end we will present some open questions and further research directions in the asymptotic theory of zeta functions.